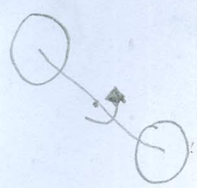
EUF 2012-1sem 27/03/2016

Q1

a)



Inercia esfera oca.

$I = \int r^2 \, dm$

$dm = \rho \, dA = \rho \cdot 2\pi r \cdot R \, d\theta$ $\sin\theta = \frac{r}{R}$

$dm = 2\pi \sin\theta R^2 d\theta \Rightarrow I = \int R^2 \sin^2\theta \sin\theta \, 2\pi R^2 \, d\theta$

$I = 2\pi R^4 \rho \int_0^{\pi} \sin^3\theta \, d\theta = 2\pi R^4 \rho \cdot \frac{4}{3} = \boxed{\frac{2R^2M}{3}}$

$L = r \times p = I\omega$

$I_T = \left(\frac{2}{3}MR^2 + \frac{Md_1^2}{2}\right) 2$ $d_1^2 = \frac{d^2}{2}$ $4R$ → dois eixos paralelos

$2 \cdot I_T = \frac{2}{3}MR^2 + \frac{16MR^2}{2} = \boxed{\frac{100MR^2}{3}} \Rightarrow \boxed{L = \frac{100MR^2}{3} \cdot \omega}$

$\omega = \frac{2\pi}{T_0}$

b) $L_i = L_f$ $d = \frac{R}{2}$

$I_T' = 2 \cdot \left(\frac{2}{3}MR^2 + \frac{MR^2}{4}\right) = \frac{22MR^2}{12} = \frac{11MR^2}{6}$

$\frac{100MR^2\omega_i}{3} = I_T'\omega_f \Rightarrow \omega_f = \frac{100MR^2\omega_i}{3} \cdot \frac{6}{11MR^2} \Rightarrow \boxed{\omega_f = \frac{300\,\omega_i}{11}}$

c) $K = \frac{1}{2}I\omega^2$

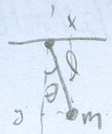
$K_i = \frac{1}{2} \cdot \left[\frac{100MR^2}{3}\right] \cdot \omega_i^2 \Rightarrow \Delta K = K_f - K_i = \frac{\omega_i^2 \cdot MR^2}{6}\left[150 - 100\right] = \boxed{\frac{25\omega_i^2 MR^2}{3}}$

$K_f = \frac{1}{2} \cdot \frac{11MR^2}{6} \cdot \omega_f^2$

d) $W = \Delta K$

Q2.

a)



$x = \sin\theta \cdot l \to \dot{x} = l \cdot \dot\theta \cos\theta$

$y = -\cos\theta \cdot l \to \dot{y} = +l\dot\theta\sin\theta$

$$T = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2) = \frac{1}{2}ml^2\dot\theta^2$$

$$U = mgh = +mgl\cos\theta \Rightarrow L = \frac{1}{2}ml^2\dot\theta^2 - mgl\cos\theta$$

$$ml^2\ddot\theta + mgl\sin\theta = 0$$

$$\boxed{\ddot\theta + \frac{g}{l}\theta = 0}$$

$x = l\theta$

$a = l\ddot\theta$

$F = ma$

$$ml\ddot\theta = \cancel{+mg\sin\theta} \Rightarrow \boxed{\ddot\theta - \frac{g}{l}\theta = 0}$$

$$\boxed{w^2 = \frac{g}{l}}$$

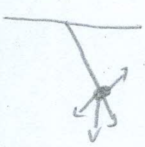
b) $r^2 = \frac{g}{l} \Rightarrow r = \pm i\sqrt{\frac{g}{l}}$

$$\theta(t) = \cancel{A\cos\left(\sqrt{\tfrac{g}{l}}t\right)} + B\sin\left(\sqrt{\tfrac{g}{l}}t\right) \Rightarrow \quad \theta(0) = 0,\ \frac{d\theta(0)}{dt} = \Omega$$

$$\dot\theta(t) = B\sqrt{\frac{g}{l}}\cos\left(\sqrt{\frac{g}{l}}t\right) \Rightarrow \boxed{B = \Omega\sqrt{\frac{l}{g}}}$$

$$\boxed{\theta(t) = \frac{\Omega}{w}\cos(wt)}$$

c)



$$-mg\sin\theta + 2m\sqrt{gl}\,\dot\theta = F \Rightarrow m \cdot a = F$$

$$ml\ddot\theta = 2m\sqrt{gl}\,\dot\theta - mg\sin\theta$$

$$\boxed{\ddot\theta - 2\sqrt{\frac{g}{l}}\dot\theta - \frac{g}{l}\theta = 0}$$

$\boxed{2}$

d)

$r^2 - 2w r - w^2 = 0$

$\Delta = 4w^2 \cdot 4w^2 = 0 \Rightarrow \boxed{r = w} \rightarrow \theta(t) = t \cdot A e^{iwt} + B e^{iwt}$

$\theta(0) = \theta_0 \quad \dot\theta(t) = A e^{iwt} + iwt A e^{iwt} + iw B e^{iwt}$

$\dot\theta(0) = 0$

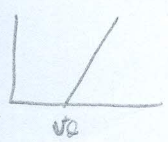
$\dot\theta(0) = 0 = A + iwB \Rightarrow A = B\, iw B$

$\theta(0) = \theta_0 = B + B \rightarrow A = -iw\theta_0$

$\Rightarrow \boxed{\theta(t) = -iw\theta_0 \cdot t e^{iwt} + \theta_0 e^{iwt}}$

Q3. 1 ms duração, $\lambda = 414$ nm, $P_0, 3P_0, 5P_0$, $P_0 = 300$ keV/s

a)



$K_{max} = e\upsilon_0 - \phi$

emissão de fotoeléctros não depende da potência, apenas da freq.

Potência aplicada $\Rightarrow P = 300 \cdot 10^3$ eV/s $\cdot 10^{-3}$ s $= 300$ eV

$$E = \frac{h \cdot c}{\lambda} = \frac{4{,}14 \cdot 10^{-15} \cdot 3 \cdot 10^8}{4{,}14 \cdot 10^{-7}} = 3 \cdot eV$$

Irá ocorrer emissão apenas p/ Li!

b) $N_{photons} = \frac{P_{laser}}{E} = \frac{\lambda}{hc} P_{laser} = \frac{900 \text{ eV}}{3 \text{ eV}} = 300$ photons sendo emitido do laser

$\rightarrow \frac{\text{Potência do laser}}{\text{"Potência" de cada photon}}$

Considerando $\eta = 1 \Rightarrow$ poderiam ser emitido 300 photons das placas de Litio.

c) $S_c \rightarrow 1s^2 2s^2 2p^6 3s^2 3p^6 4s^2 3d^1$ $\quad 4s^2 \quad 3d$

w/l... $\quad n=4 \quad n=3$

$l=0 \quad l=2$

d) $L_z = m\hbar$

$m = 0, \pm 1, \pm 2, \pm l \quad \Rightarrow \quad 3d^1 \; n=3 \; l=2 \quad \Rightarrow \quad \boxed{L_z = -2\hbar, -\hbar, 0, \hbar, 2\hbar}$

$L = \sqrt{l(l+1)}\,\hbar \Rightarrow \quad \boxed{L = \sqrt{6}\,\hbar}$

Q4.

a) $\frac{\partial^2 \psi}{\partial x^2} + k^2 \psi(x) = 0 \quad \rightarrow \quad k^2 = \frac{2mE}{\hbar^2}$ $\quad \psi(0) = 0 \quad \psi'(0) = 0$

$\psi(d) = 0 \quad \psi'(d) = 0$

b) $\psi(x) = A\sin(kx) + B\cos(kx)$

$A\sin(kd) = 0 \Rightarrow \boxed{k = \frac{n\pi}{d}} \Rightarrow \boxed{\psi(x) = \sqrt{\frac{2}{d}} \sin\left(\frac{n\pi}{d}x\right)}$

$\int_0^d A^2 \sin^2(kx)\,dx = 1 \Rightarrow \frac{A^2 d}{2} = 1 \Rightarrow A = \sqrt{\frac{2}{d}}$ $\quad E_n = \frac{n^2\pi^2\hbar^2}{d^2 2m}$

c) $\psi(x) = \sqrt{\frac{2}{d}} \sin\left(\frac{3\pi x}{d}\right)$; $\boxed{n=3} \rightarrow \boxed{E_3 = \frac{9\pi^2\hbar^2}{2md^2}} \rightarrow E_3 = \frac{hc}{\lambda} \Rightarrow \boxed{\lambda = \frac{hc}{E_3}}$

d) $P(0<x<\frac{d}{6}) = \int_0^{d/6} |\psi(x)|^2 dx = \frac{2}{d}\int_0^{d/6} \sin^2\left(\frac{3\pi x}{d}\right) dx = \frac{1}{d}\left[\frac{d}{6} - \frac{d}{6\pi}\cdot(\sin(\pi) - \sin(0))\right]$

4 $\quad \boxed{= \frac{1}{6}}$

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Q6. $a<r<b$ $r>b$

a) $B = \frac{\mu_0 I}{2\pi r}$ b) $B = 0$

c) $B = \frac{\mu_0 I_{enc}}{2\pi r}$ $I_{enc} = \frac{4\pi r^2}{4\pi a^2} \cdot I$ $\Rightarrow$ $\boxed{B = \frac{\mu_0 r I}{2\pi a^2}}$

$H = \frac{B}{\mu}$

d) $w_B = \frac{1}{2}\mu H^2 = \frac{1}{2}\frac{B^2}{\mu} = \frac{1}{2\mu}\left(\frac{\mu_0 I}{2\pi r}\right)^2 = \frac{\mu_0 I^2}{8\pi^2 r^2}$

$U_B = \iiint w_B \, dV = \int_a^b \frac{\mu_0 I^2}{8\pi^2 r^2} 2\pi r l \, dr = \frac{2\pi \mu_0 l}{8\pi^2} \ln\left(\frac{b}{a}\right)$

$\boxed{\frac{U_B}{l} = \frac{\mu_0 \ln\left(\frac{b}{a}\right)}{4\pi}}$

Q7.

a) metade superior

$E = \frac{Q}{4\pi r^2 \varepsilon_0}$ $\kappa = \frac{\varepsilon}{\varepsilon_0}$

inf.

$E = \frac{Q}{4\pi r^2 \kappa \varepsilon_0}$

b) $V = \frac{Q}{4\pi r \varepsilon}$ $\Rightarrow$ $D = \frac{Q}{4\pi r^2}$ $\sigma_l 4\pi a^2 = Q$ $\nabla \cdot D = \rho_l$

$\rho_l = 0$

$\sigma_l = \frac{Q}{4\pi a^2}$ $= -\frac{Q}{4\pi b^2}$

c) $\sigma_b = P \cdot \hat{n}$ $P = \varepsilon_0 \chi_e E = \frac{\varepsilon_0 \chi_e \cdot Q}{4\pi r^2 \varepsilon_0}$

$$\sigma_b = \begin{cases} \frac{\chi_e Q}{4\pi a^2} \hat{r} \\ -\frac{\chi_e Q}{4\pi b^2} \hat{r} \end{cases}$$

d) a densidade de carga de polarização sobre a superfície plana do dielétrico é nula porque $\hat{z} \perp \hat{r}$

e) $C = \frac{Q}{V_A - V_B} = \frac{Q}{\frac{Q}{4\pi\varepsilon a} - \frac{Q}{4\pi\varepsilon b}} = \frac{4\pi\varepsilon (r_a r_b)}{(r_b - r_a)}$

Q8. a) $\boxed{a\Psi_0 = 0}$

$$\sqrt{\frac{m\omega}{\hbar}}\, x\,\Psi_0 + \sqrt{\frac{\hbar}{m\omega}}\,\frac{d\Psi_0}{dx} = 0$$

$$\frac{d\Psi_0}{dx} + \frac{m\omega}{\hbar}\Psi_0 \cdot x = 0 \Rightarrow \frac{d\Psi_0}{dx} = -\frac{m\omega x}{\hbar}\Psi_0$$

$$\frac{d\Psi_0}{\Psi_0} = \frac{-m\omega x\,dx}{\hbar} \Rightarrow \ln\Psi_0 = \frac{-m\omega x^2}{2\hbar}$$

b) $$\int_{-\infty}^{\infty} A^2 e^{\frac{-m\omega x^2}{\hbar}}\, dx = 1$$ $$\boxed{\Psi_0 = A\, e^{\frac{-m\omega x^2}{2\hbar}}}$$

$\alpha = \frac{m\omega}{\hbar}$

$$A^2 \cdot \left(\frac{\pi\hbar}{m\omega}\right)^{\frac{1}{2}} = 1 \Rightarrow \boxed{A = \left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}}}$$

c) $$\Psi_0' = \frac{-m\omega x}{\hbar} A\, e^{\frac{-m\omega x^2}{2\hbar}}$$

$$\Psi_0'' = \frac{-m\omega}{\hbar} A\, e^{\frac{-m\omega x^2}{2\hbar}} + \frac{m\omega x}{\hbar}\cdot\left(\frac{m\omega x}{\hbar}\right) \cdot A\, e^{\frac{-m\omega x^2}{2\hbar}}$$

$$\frac{-\hbar^2}{2m}\left[\frac{-m\omega \cancel{A}}{\hbar}\cancel{e^{-\beta x^2}} + \left(\frac{m\omega x}{\hbar}\right)^2 \cancel{A e^{-\beta x^2}}\right] + \frac{1}{2} m\omega^2 x^2 \cdot \left(\cancel{A e^{-\beta x^2}}\right) = E_0 \cdot \left(\cancel{A e^{-\beta x^2}}\right)$$

$$E_0 = \frac{\hbar\omega}{2} - \cancel{\frac{m\omega^2 x^2}{2}} + \cancel{\frac{1}{2} m\omega^2 x^2} \Rightarrow \boxed{E_0 = \frac{\hbar\omega}{2}}$$

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d) $V(x) = V_0 \exp\left(-\frac{x^2}{b^2}\right)$

$$E_0^{(1)} = \langle \psi_0 | V | \psi_0 \rangle = \int_{-\infty}^{\infty} A^2 \cdot V_0 e^{-\frac{x^2}{b^2}} \cdot e^{-\frac{m\omega x^2}{2\hbar}} dx$$

$$E_0^{(1)} = A^2 V_0 \int_{-\infty}^{\infty} e^{x^2\left(-\frac{1}{b^2} - \frac{m\omega}{2\hbar}\right)} dx \quad \left(-\frac{1}{b^2} - \frac{m\omega}{2\hbar} \rightarrow \frac{-2\hbar - m\omega b^2}{2\hbar b^2}\right) \Rightarrow E_0^{(1)} = A^2 V_0 \cdot \left(\frac{\pi}{\alpha}\right)^{\frac{1}{2}} \quad \alpha = \frac{2\hbar + m\omega b^2}{2\hbar b^2}$$

$$E_0^{(1)} = \left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{2}} \cdot \left(\frac{\pi 2\hbar b^2}{2\hbar + m\omega b^2}\right)^{\frac{1}{2}} \cdot V_0 = \boxed{\left(\frac{2m\omega b^2}{2\hbar + m\omega b^2}\right)^{\frac{1}{2}} \cdot V_0}$$ deslocamento de energia.

Energia total

$$\boxed{E_0 = E_0^{(0)} + E_0^{(1)}}$$

Q5.

a) $\vec{\mu} = \gamma \vec{S}$

$$H = -\vec{\mu} \cdot \vec{B} = -\gamma \vec{B} \cdot \vec{S} \Rightarrow H_z = -\gamma \vec{B} \vec{S}_z = -\gamma \vec{B} \frac{\hbar}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

b) $\hat{H}|\psi\rangle = E_n |\psi\rangle$

$$-\gamma \vec{B} \frac{\hbar}{2} \begin{vmatrix} 1-\lambda & 0 \\ 0 & -1-\lambda \end{vmatrix} = 0 \Rightarrow \lambda = \mp \gamma \vec{B} \frac{\hbar}{2}$$

$$\boxed{E_n = \pm \gamma \vec{B} \frac{\hbar}{2}}$$

$\lambda_1 = -\gamma \vec{B} \frac{\hbar}{2}$

$$\begin{bmatrix} -\gamma \vec{B} \frac{\hbar}{2} & 0 \\ 0 & \gamma \vec{B} \frac{\hbar}{2} \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = -\gamma \vec{B} \frac{\hbar}{2} \begin{bmatrix} a \\ b \end{bmatrix}$$

$$-\gamma \vec{B} \frac{\hbar}{2} a = -\gamma \vec{B} \frac{\hbar}{2} a \Rightarrow |E_1\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\gamma \vec{B} \frac{\hbar}{2} b = -\gamma \vec{B} \frac{\hbar}{2} b$$

$$|E_2\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

c) $\chi(0) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ e^{i\alpha} \end{pmatrix} = \underbrace{\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ e^{i\alpha} \end{pmatrix}} = \frac{1}{\sqrt{2}}|E_1\rangle + \frac{e^{-i\alpha}}{\sqrt{2}}|E_2\rangle$

$i\hbar \frac{\partial \Psi(x,t)}{\partial t} = H\Psi(x,t) \Rightarrow \chi(t) = e^{-\frac{iHt}{\hbar}} \cdot \chi(0)$

$$\boxed{\chi(t) = \frac{e^{\frac{i\gamma B t}{2}}}{\sqrt{2}}|E_1\rangle + \frac{e^{-i\alpha} \cdot e^{-\frac{i\gamma B t}{2}}}{\sqrt{2}}|E_2\rangle}$$ ou

$i\hbar\begin{pmatrix} \dot{c}_1 \\ \dot{c}_2 \end{pmatrix} = \begin{pmatrix} -\frac{\gamma B\hbar}{2} & 0 \\ 0 & +\frac{\gamma B\hbar}{2} \end{pmatrix} \cdot \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$

$i\hbar \dot{c}_1 = -\frac{\gamma B\hbar}{2} c_1 \rightsquigarrow \frac{dc_1}{c_1} = \frac{i\gamma B}{2} dt \Rightarrow \boxed{c_1 = Ae^{\frac{i\gamma B}{2}t}}$

$i\hbar \dot{c}_2 = \frac{\gamma B\hbar}{2} c_2 \rightsquigarrow \frac{dc_2}{c_2} = -\frac{i\gamma B}{2} dt \Rightarrow \boxed{c_2 = Be^{-\frac{i\gamma B}{2}t}}$

$|\Psi(t)\rangle = Ae^{\frac{i\gamma B t}{2}}|+a\rangle + Be^{-\frac{i\gamma B t}{2}}|-a\rangle$

$|\Psi(0)\rangle = A|+a\rangle + B|-a\rangle \Rightarrow A = \frac{1}{\sqrt{2}}, \; B = \frac{e^{-i\alpha}}{\sqrt{2}}$ //

d) $S_x = \frac{1}{\sqrt{2}}|E_1\rangle + \frac{1}{\sqrt{2}}|E_2\rangle \quad \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = 1\begin{bmatrix} a \\ b \end{bmatrix} \Rightarrow b = a \begin{bmatrix} 1 \\ 1 \end{bmatrix} \Rightarrow |+_x\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$|\langle S_x \tfrac{\hbar}{2}|\Psi(t)\rangle|^2 \quad \left|\frac{1}{2}\left[\langle E_1| + \langle E_2|\right] \cdot \left[e^{\frac{i\gamma B t}{2}}|E_1\rangle + e^{-i\left(\frac{\gamma B t}{2} + \alpha\right)}|E_2\rangle\right]\right|^2$

$= \left|\frac{1}{2}\left[e^{\frac{i\gamma B t}{2}} + e^{-i\left(\frac{\gamma B t}{2} + \alpha\right)}\right]\right|^2 = \frac{1}{4}\left[e^{\frac{i\gamma B t}{2}} + e^{-\frac{i\gamma B t}{2}} \cdot e^{-i\alpha}\right]\left[e^{-\frac{i\gamma B t}{2}} + e^{\frac{i\gamma B t}{2}} \cdot e^{i\alpha}\right]$

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$\phi_1 = \frac{i\gamma B t}{2}$ $\quad \cos^2 x = \frac{(1+\cos 2x)}{2}$

$\phi_2 = -\frac{i\beta t \gamma}{2} - i\alpha$

$$P_+(t) = \frac{1}{4}\cdot\left(1+1+e^{i\gamma B t}\cdot e^{i\alpha} + e^{-i\gamma B t}\cdot e^{-i\alpha}\right)$$

$$= \frac{1}{4}\left[2+\left(e^{i(\alpha+\gamma B t)} + e^{-i(\alpha+\gamma B t)}\right)\right]$$

$2\cdot\cos(\alpha+\gamma\beta t) = 2\cos^2 x = 1+\cos 2x$

$$= \frac{1}{2}\left[1+\cos(\gamma B t+\alpha)\right] = \frac{1}{2}\cdot\left[2\cdot\cos^2\left(\frac{\gamma B t+\alpha}{2}\right)\right]$$

$$\boxed{= \cos^2\left(\frac{\gamma B t+\alpha}{2}\right)}$$

Q11.

$P_0 = \frac{nRT_i}{V_0}$ $\quad P_f = \frac{2nRT_f}{V_0}$ $\quad P_0 V_0 = P_f V_f$

$\boxed{P_f = 2P_0}$

a) $dU = \cancel{dQ} + dW$

$\Delta U = -W \to \Delta U = -p\,dV$

$$\Delta U = \int_{V_0}^{\frac{V_0}{2}} \frac{dV}{V} nRT = nRT \ln\left(\frac{1}{2}\right)$$

b) $Q = ST$

$S = \frac{Q}{T} \Rightarrow \Delta S = 0$

voltar